ORIGINALS IN GEOMETRY.

By Harry B. Marsh.

In dealing with originals in geometry, or in any other branch of knowledge or activity, we can safely assume that it is human nature to like originals, to desire to make use of whatever information may be at hand in order to branch out along new and independent lines. This phase of human nature has played no little part in starting the great explorers of all times upon their epoch making expeditions; and who can deny that the discovery of this continent was due to the supreme effort of Christopher Columbus to demonstrate a world-embracing original in spherical geometry. This eagerness for individual achievement and success has been the inspiration of leaders and inventors, and it has been a very effective force back of all progress. It is present to a greater or less extent in all human beings.

By the time boys and girls have reached high school age, their liking to do original things is well developed and very evident. We see it displayed, for instance, in the manual training and domestic science classes, where the pupils listen to the directions of the instructor concerning the use of the various tools and implements employed, but all the while are impatient to begin doing and making things themselves. Is it logical to believe that this liking for originals is limited to the field of manual arts and sciences? By several tests with groups of boys the writer has satisfied himself that the natural liking for original work can be applied as well to mental as to manual work; to geometry as to carpentry. What is more important, he has found that he gets much better results by assuming that pupils ought to and do like originals than he ever could by thinking that they ought not to and do not.

The teaching of originals begins with the study of formal geometry, for the theorems and corollaries and definitions are the

(*Note.*—When this talk was given, all the points and methods brought out were illustrated by many figures which had been placed upon the board. Hence the following can serve but as an outline.)

working tools of geometry. The carpenter does not tell the apprentice simply that the chisel is a very sharp instrument of high grade steel. He explains that it is a handy tool to have when this or that piece of work is to be done. So, as we take up the different propositions, we should explain their uses and applications to certain kinds of originals. This can be done without breaking into the continuity of the subject, by introducing originals after small groups of theorems. After such a proposition as, "Upon a given line as a chord, to describe a segment of a circle in which a given angle may be inscribed," it is well to let the class know that this is often a handy construction to use when a triangle is to be built with the base and the angle opposite the base as parts of the given. It also has many applications in locus problems which have to do with a fixed angle. Then an exercise such as (C. E. B., 1908): "Let A and B be two fixed points on the circumference of a circle and P and Q the ends of a variable diameter of the same circle. Find the locus of the point of intersection of the straight lines PA and QB," furnishes an excellent application of this proposition and the ones that have preceded it in the measurement of angles. Many other theorems or groups can be treated in the same way, and made a valuable preparation for original work. Thus the pupils are made to feel that the relationship between theorems and originals is a close one.

When we consider originals themselves, we should keep in mind that our chief problem is not only to increase and encourage the natural liking of our pupils for them but still more important, not to diminish and discourage it. When once a boy looks upon all originals as hopeless, he is very likely to consider them as entirely beyond him and to make very slight attempt at solving them. The first and foremost cause of such discouragement comes from poor grading. If a class is led through a series of exercises which grow more difficult as fast as, but no faster than, their ability increases, they will attack problems that require no little mental effort with interest and confidence.

Originals should be looked upon as a pleasant break in the routine of formal theorems. They should be welcomed by both teacher and pupils alike. The attitude of the class towards them is generally a reflection of the attitude of the teacher. In order to be taken up with interest and enthusiasm by the pupils they

must be presented with equal or greater enthusiasm by the teacher. Even though, in the early months of the subject, most of the originals are taken from the text-book, it is possible to avoid a cut-and-dried assignment of them. In the latter part of first year geometry, however, and especially in review geometry, a great opportunity is offered in the line of practical problems and college entrance originals. "Here's a good one from Harvard," or, "See what you can do with this Yale question," produces at once both interest and inspiration. Such originals seem much more alive than those that have been lying dormant in some text-book for several years. In college preparatory review geometry divisions at least eighty per cent. of the originals assigned should be from recent entrance examination questions.

In teaching originals we shall do well to remember that we are trying to give the pupils a bird's-eye-view of the application of geometry. We should teach general methols; not simply special exercises. When we explain a difficult original to a class, it is more important to tell them how we know how to do it, than just how to do it, for we are dealing not only with that exercise, which the class may never have again, but with methods for similar originals which they are sure to have again. When an original is given to us our minds work along definite lines. Certain conditions in a hypothesis marshall up definite geometrical facts. The pupils should be given the advantage of this experience. They should be trained in right thinking. It is well to spend frequently a part of a period or a whole period in such drill work. Such questions as: How do you prove the products of two lines equal to the products of two others? By what means do you get a proportion? What are several ways of getting lines equal? lines parallel? angles equal? What kinds of loci do we have in plane geometry? What can you always do with an altitude in a construction? and many other similar ones, group together certain related facts in a useful way. Parallel lines not only suggest equal arcs to us, but also the substitution of one of the arcs for the other in the measurement of an angle. In this way we can frequently get two angles equal easily. In locus problems we generally try two or three special positions of the lines or points in question and get a very good indication of the solution; and so on. These are but illustrations

of many methods we have of attacking originals. The class should have the advantage of our experience and be taught and drilled on all these methods.

To emphasize and illustrate methods of solving originals, it is of advantage to take up many of them in class. In doing this our own figures should be accurate and general, if we wish to get the same kind from the class. If the conditions given should make angle A twice angle D, the figure should also do this. Otherwise, imagination as well as logic enters into the proof. The average boy or girl finds it hard enough to solve a difficult original without being handicapped. A poor figure is a big and a needless handicap.

When the exercise has been read or written on the board, all the hypothesis should be carefully studied, for a great many times failure to get an exercise is due to failure to use all of the given, every fact of which enters somewhere into the proof. Then if the solution of the exercise does not suggest itself right away have the class focus the attention upon the "To Prove," or "To Construct," rounding up all the possible methods of solution and then narrowing them down to the most probable. Check the tendency to introduce construction lines unless they are absolutely necessary, for many times they hinder rather than help a proof. Now ask for suggestions. There should be such a feeling of confidence and co-operation in the classroom that none should hesitate to propose his method. No suggestion should be rejected or accepted by the teacher without showing why it was or was not of use. By working together in this way occasionally, the pupils, if only from their habits of imitation, get hold of methods that are worth while.

The writer has also found that he gets better results if he supplements the home work on originals by frequent written class exercises. In these the pupil is not only thrown absolutely upon his own resources, but he is also compelled to so concentrate his mind that he can get the solution in a certain specified time. Numerous class exercises of this kind furnish excellent drill work for any who are planning to take entrance examinatione and it would be hard to find a satisfactory substitute for them. A division that has been made to write out many originals in class will be found to be much better trained and better equipped than one that has not had this practice.

In closing let me urge the importance of holding up the whole recitation for the boy or girl who has a different, and oftimes what proves to be a better proof than the one that has been given for an exercise. That boy may have worked hard and long the night before, and when he finally mastered the original he had earned not only the joy that comes from the successful completion of such a task but also the right to show his proof the next day in class. Everything else is worth giving up for the five or ten minutes the boy will use, for we are not only justly rewarding him for his efforts but we are furnishing an incentive and inspiration to the rest of the class to try to do as well.

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